Rule of Thumb Proposing the Size of Aperture Expected to be Sufficient to Resolve Double Stars with Given Parameters

Wilfried R.A. Knapp

Vienna, Austria
wilfried.knapp@gmail.com

Abstract: Visual observation of double stars is an anachronistic passion especially attractive for amateurs looking for sky objects suitable for visual observation even in light polluted areas. Session planning then requires a basic idea which objects might be suitable for a given equipment—this question is a long term issue for visual double star observers and obviously not easy to answer, especially for unequal bright components. Based on a reasonably large database with limited aperture observations (done with variable aperture equipment iris diaphragm or aperture masks) a heuristic approach is used to derive a statistically well founded Rule of Thumb formula.

Introduction

Visual observation of double stars is an anachronistic, if fascinating pastime for amateur astronomers; to resolve a single point of light into two or more points with the help of a telescope by increasing magnification is a gratifying experience re-enacting discovery. Double stars are attractive targets even with heavy light pollution or a full moon, as the resolution of double stars does not require a black sky—most important for backyard astronomers. With visual double star observation eventually questions like "What are the best suited targets for my telescope" or "Which aperture is needed to resolve this specific double star" arise, at least this is what happened to me when I started to observe double stars several years ago. After trying the first simple approaches, for example applying ΔM on top of Dawes or Rayleigh, it quickly became obvious that there needs to be more to give a satisfying answer to this question.

As starting point, it seems necessary to define the term "resolution", in the most strict sense this means seeing two star disks clearly separated. But we all know that this is not the end but just the best case; notched rod, rod, elongation, etc. are also to be considered. Finally, I came to the conclusion that any observation allowing a well-educated estimation of position angle and separation is to be considered as resolution. Another issue is the question if resolution means to detect a double star by simply looking at it without knowing that this might be a double or if we know in advance that the object we look at is a double star with known separation and magnitudes of components. In this report, the latter is assumed but with position angle unknown to be checked later on if correctly estimated during observation to counter-check for false positives.

Further Research

Looking back in the history of double star observing and into the optical theories regarding resolution of two close sources of light, there is a large number of serious attempts to answer this question. Without going too much into historical details the following main approaches are listed as follows:

- Rayleigh criterion: Resolution for equal bright doubles is determined by the separation, where the first diffraction minimum of the primary coincides with the maximum of the secondary. As the size of the diffraction pattern depends not only on the aperture but also on the wavelength of the light we have to settle for an average wavelength of yellow light to get a number for this diffraction-limit relationship with an aperture in mm of 138/separation in arcseconds.
- Dawes criterion: Two stars of ~6th magnitude should be resolved with an aperture in mm of 116/separation in arcseconds moderate favourable seeing provided (Dawes in 1867 according to Argyle 2012 page 107). A rather curious side aspect of this

criterion (often misleadingly termed "limit" – as we will see there is no such thing as a resolution "limit" as too many factors are involved) is the fact that the number of ~6th magnitude equal bright pairs seems statistically rather small to come to such a conclusion. Most probably, we can go very well to equal but brighter than magnitude 6 stars; this might then cause some overlap of the star disks, but still allow a clear identification as double star. This criterion is well accepted in the astronomical world and most telescope producers use it in their technical specification as "resolution limit". But we have to keep in mind that while this criterion is of good use it is also very limited.

- Lord's Rule of Thumb according to his paper "A report upon the analysis of the telescopic resolution of double stars of unequal brightness" (1994); certainly a noticeable intellectual effort but the results have to be taken with caveats. The main weakness is the strong ΔM focus leading to the absurd consequence that the same ΔM brings the same result regardless of the magnitudes of the components, what is obviously simply wrong.
- Napier-Munn makes an interesting attempt in "A Mathematical Model to Predict the Resolution of Double Stars by Amateurs and Their Telescopes" (2008) to provide a resolution probability for given double star parameters and a given aperture based on statistical analysis of a rather small number of observations and not covering apertures below 80mm.
- Argüelles (2001, Fuzzy Splitting) uses an interesting fuzzy logic approach to calculate a so called Difficulty Index for splitting double stars but without any relation to a given or required aperture. The value of such an information seems at first impression a bit questionable, but values of this index near 100 simply indicate that even with a large aperture you need really excellent seeing for resolution.
- There are, without doubt, many other double star observers with noticeable efforts on this topic (for example Treanor, Lewis, Markowitz to name a few), but as this report is not intended as a historical research the listed main attempts should be a sufficient base for further investigations.

What is obvious from the very beginning is the fact that a purely analytical approach is doomed to fail. It seems impossible to take all relevant factors into account, not only for their number but also for the lack of reliable data:

- Most surprising, and often overlooked, is the fact that even the basic double star parameters such as the magnitudes of the components are often quite unreliable to an unexpected degree of up to 2 or more magnitudes (think Jonckheere).
- In the same line but for other reasons we have the class of variable stars.
- Even seemingly reliable separations might be tricky, either because the orbits are known or unknown, or for their lack of precision when it comes to very close pairs. It makes a huge difference if the separation is 0.65 or 0.74 arcseconds, but WDS usually lists both cases as simply 0.7".
- Color issues: Given magnitudes might be precise to the second digit after the decimal point yet for visual resolution it makes a huge difference if both components are blueish white or reddish yellow or in worst case the primary is blueish white and the secondary is reddish yellow. But star color does not only effect the human eye perception but also the size of the diffraction pattern, especially diameter of the central disk.
- Seeing: The effects of seeing on the resolution of double stars are anything but straightforward. Even small variations in seeing conditions (basically defined as the degree of solid and stable resolution of the diffraction pattern = the bull's-eye optical artifact of a light source when we are looking at through a telescope) can make a huge difference at least for close pairs. The difference between fair and truly excellent seeing covers several aperture classes and is cause for a good part of the recorded spread of apertures for the very same doubles or such with very similar parameters. Basic requirement for a reasonable Rule of Thumb is fair seeing meaning that at least the spurious star disk (central part of the diffraction pattern) is crisp and stable if only for fractions of seconds – this means something around ~5 on the Pickering seeing scale. At the same time experience has shown that trying to assess the seeing quality according to Pickering or any other scheme does not really help – minor differences not represented by a single number of a scheme might make all the difference when it comes to observe at the aperture limit. So the for me most useful procedure to assess fair seeing is to include a benchmark object in each session plan in form of an equal bright ~6mag double star with a separation between Dawes and Rayleigh criterion. Most basic example would be both Eps Lyr doubles with a 60mm telescope – a clear resolution would indicate fair seeing. On the other side the effects of less than fair seeing are negligible for wide pairs but disastrous for close pairs to the degree of non-resolution regardless aperture as bad seeing cannot

be compensated by using a larger apertures – increasing the size of aperture even increases the negative effects of bad seeing. Session planning should take these effects into account by including a fair amount of wide pairs for such conditions to avoid a completely frustrating experience.

- Light pollution: Double star observing is surprisingly robust against light pollution with the exception of very faint stars near the telescope magnitude limit – with heavy light pollution we might lose even more than one magnitude in TML but with little effect on the brighter stars. Experiments in this area have shown that doubles with companions brighter than +10mag show little to no impact of light pollution.
- Multiples: Faint components between two brighter ones are certainly harder to resolve than simple doubles.
- Star field characteristics: The resolution of doubles might get difficult with bright stars in the field of view especially with rather faint components.
- Equipment: Quality and design of optics. Refractor or reflector and which kind. For the latter the size of central obstruction and collimation seem especially important.
- Observer: Experience and routine in observing double stars (using for example techniques like averted vision or moving the target through the field of view).
- Personal acuity of the observer: The eyes and the brain of the observer are essential parts of the optical system. The personal acuity is one of the parameters of this system and is often expressed as visual angular separation in arcseconds needed to resolve two equal bright light sources corresponding with the factor of magnification (given by Acuity/Separation) needed for resolution when using a suitable telescope.

To get a better grip on this topic I started to make limited aperture observations using telescopes with variable aperture by either aperture masks or aperture iris with preference for the latter because it works continuously and not in steps and is less cumbersome to use (see Appendix image 30). This was done by making session plans with objects assumed to be suitable for the telescope in use, starting the session by assessing the current naked eye magnitude limit and the seeing conditions for the sky area in question, locating the objects by star hopping or with the help of a DSC (Digital Setting Circle) system, resolving the double if possible under the given conditions, noting the details for later on checking if correctly resolved and reducing then the aperture down to the degree of non-resolution and noting the aperture with resolution just possible together with the required magnification. After checking all details after the session I added these observation details to an ever growing data set of limit observations. Already in a very early stage of this process two things got clear:

- There is no such thing like a single limit aperture number. When I did observations on the same objects in different sessions I got different results even if the conditions seemed quite similar. For the very first of such cases I deleted the results with the larger aperture looking for "the" limit aperture but soon I realized that one of the consequences of these at least partly unpredictable factors is that we can never derive a definite fixed number as so called "limit" - the best we can aim for is a reasonable usable Rule of Thumb with some unavoidable spread as different observation sessions will provide different results.
- A relatively large number of objects yielded rather unexpected results with non-resolution where it should have been easy and the other way around with resolution far easier than expected – so it got evident that the data derived from the WDS catalog was to some degree questionable, especially the magnitudes.

This led to two consequences: I recorded all limit aperture observation results even if different for the same object and I started to look for ways to counter-check the WDS data for suspect objects. The latter was in a first step done by checking other star catalogs like APASS, UCAC4, Nomad, URAT1 etc. for useful V-mag data or other hints like calculated estimations from magnitudes in other bands like J- and K-band values. This led in communication with Brian Mason/USNO to WDS Vmag estimation corrections for many objects. Yet this procedure was not fully satisfying as estimations were replaced again by estimations if probably better ones so I was on the lookout for doing better.

The question of the data structure of the limit aperture observations data set was decided pragmatically based on the situation given with session planning means data usually available before starting the session:

- Basic double star parameters like separation in arcseconds and magnitudes of the components as given in the WDS catalog but counter-checked with additional information like for example known orbits
- Aperture and size of central obstruction (the latter zero for refractors and quota of aperture diameter for reflectors)

- Naked eye magnitude limit as indication for the degree of light pollution assuming that each location has in advance some average expectations in this regard.

All other relevant parameters were assumed to be as follows:

- At least fair seeing is what you can hope for but this cannot be part of the planning
- Average experience and acuity of the observer (at least this is my self-assessment)
- Spectral class information is currently not generally available for both components of doubles so of little use for session planning.

In the beginning of the data sampling process I also recorded observations from other observers with reliable reputation, but such reports were usually based on telescopes with fixed aperture and exceptional good seeing conditions providing a bias towards "over-performance" and not really good usable when looking for the rule and not the exception. So eventually I deleted all records form other observers and kept only my own, but with a small number of observations at the fixed apertures of the other telescopes when further reduction of the aperture did not seem suitable. The following telescopes were used in this project:

- Refractors of 80, 120, 140 and 185mm aperture
- Reflectors of different type with 200 and 235mm aperture.

The use of the mentioned reflectors in this project was of rather little benefit for several reasons, one of them the rather large initial size of central obstruction making the concept of variable aperture obsolete and the other the simple fact that you have to have good enough seeing to benefit from the larger aperture; rather rare in my locations.

My project plan was to cover the range of apertures as far as possible within the range of telescopes available to me but in the end I had to accept that the fair seeing requirement is a moving target depending on the aperture size used with the consequence that the number of recorded limited aperture observations in the range of 150 to 200mm is smaller than hoped for and to some degree may be also the side effect of especially bad seeing conditions in 2016/2017 with often several weeks of cloudy nights between observation sessions. See Table 1 for results of these observations.

An analytical answer to the RoT question would probably show a nested if-then-else structure beginning with the basic assumption of equal bright components in the 6mag range and the Dawes criterion with 116/sep

Table 1: Examples of limit aperture observations done in the last years with variable aperture telescopes

Object	Aperture (mm)	Sep	M1	M2	NEML	CO
STF2786	55	2.80	7.49	8.20	2.8	0
STF2786	60	2.80	7.49	8.20	3.5	0
STF2791	70	2.80	8.92	9.28	4.7	0
STF2791	75	2.80	8.92	9.28	3.5	0
STF2792	85	7.20	9.20	10.60	4.5	0
STF2795	95	1.70	9.31	9.66	2.8	0
STF2795	105	1.70	9.31	9.66	2.8	0
STF2795	110	1.70	9.31	9.66	2	0
STF2795	140	1.70	9.31	9.66	2.8	0.35
STF2810	35	17.00	8.43	9.04	2.8	0
STF2812	80	2.30	9.25	9.70	2.8	0
STF2813	50	10.40	9.21	9.72	2.8	0
STF 282	62	6.70	9.47	9.51	2.65	0
STF2822	65	1.80	4.80	6.20	4.5	0
STF2822	90	1.80	4.80	6.20	3	0

(separation in arc-seconds) giving the proposed limit aperture diameter in mm assuming fair seeing. After sampling my first ~200 limit aperture observations I did a first attempt for finding a reasonable RoT formula based on statistical analysis of the given data set using a step by step approach for developing an algorithm and a statistical program XLSTAT to derive parameter values by nonlinear regression.

My first attempts of 2013 began by combining educated guesses and trial and error by best adapting to the existing data and were based on the structure: rA = required Aperture in mm = rA1+rA2+rA3+rA4+rA5 with a resolution probability of 50% with

- rA1 = Dawes criterion as starting point = 116/Sep with Sep = separation in arcseconds. It would certainly be possible so start with Rayleigh or other criterions but from personal experience Dawes seems a good choice especially as this criterion is kind of standard in the technical specification of telescopes.
- rA2 = f(ΔM) = function of magnitude difference in relation to separation = P1*(ΔM)/Sep^P2 with P1 and P2 as parameters to be determined by nonlinear regression. This part of the function assumes that resolution gets ever more difficult with increasing difference M2-M1 combined with decreasing separation while ΔM of less than 1mag might considered to be without effect.
- rA3 = f(M1) = function of M1 representing the fact

that resolution gets more difficult with decreasing brightness of the primary like for example 7/10mag pair compared with 4/7mag pair. This might start with a magnitude of 6 as base of the Dawes criterion and is to be seen in relation with separation = P3*(M1-P4)/Sep^-P5 with P3, P4 and P5 again as parameters to be determined by nonlinear regression.

- rA4 = f(M2) = function of M2 assuming that resolution gets additionally more difficult with fainter companions = P6*(M2-P7) working only for M2 fainter than 9mag with P6 and P7 again as parameters to be determined by nonlinear regression. This function might be considered redundant as this aspect seems already covered by M1 and ΔM but on the other side using M2 separately adds more flexibility to statistical analysis and gets higher in relevance with increasing faintness of M2 where we have to keep an eye on the telescope magnitude limit for a given aperture to avoid an invalid result.
- rA5 = f(NEML) = function of NEML = P8*(6.5-NEML)^P9/6.5 with NEML = assumed naked eye magnitude limit for a given location with 6.5mag for a perfect black sky. Light pollution has as already mentioned surprisingly little effect on double star resolution – but some if small influence on the required aperture is certainly given and has for this reason to be included in a reasonable RoT formula. What is still missing here is the already mentioned fact that light pollution is of little to no influence when resolving bright pairs but of probably significant influence for fainter components.

Already at this early stage the statistical analysis worked rather well for a reasonable range of apertures and double star parameters. This motivated me to proceed further and to add parameters for the effect of central obstruction size, to have a closer look at the topic of telescope magnitude limit for a given aperture and to look at outlier results for enhancing the RoT formula.

- Size of the central obstruction: One effect of the size of central obstruction on the diffraction pattern is, according to Mahajan 2011, a reduction of the size of the central disk (to be precise: first minimum) of about 20% from zero to 0.5 combined with a reduction of the encircled energy of about 43% (see Appendix Table 1). This means that we should have, at least for brighter stars, a small advantage for resolving double stars due to the smaller central disks. To represent this effect I simply added a corresponding factor to the Dawes criterion representing the reduction in size of the central disk of the diffraction pattern. The fact that increasing CO above some threshold would be counterproductive I left to be solved by statistical analysis by introducing an additional parameter for this effect. But at this stage it was clear it required additional experiments to get an idea how this really works but this was scheduled as a future task.
- Telescope magnitude limit: To avoid getting results with an aperture too small to be able to resolve a faint companion regardless if double of not, I decided to add an extra step with a counter-check. What sounds easy proved to be far more complicated than anticipated, because the task to determine the TML for a given aperture seems to be a quite complex one as demonstrated by the work of Schaefer 1990 on this topic (see also the online implementations from Bogan 1998 and Houdart 2008). Despite a multitude of influencing parameters the final counter-check with real observations (Schaefer 1990) resulted in a considerable spread in the results, rendering his complex analytical approach to being not much more useful than the often used crude formula 2.7+5*Log10(mm) with mm for the aperture diameter in mm. Besides, the results of Schaefer's model seem overall rather too optimistic when compared with the technical specifications of telescope producers having certainly no reason to be overly modest. The bad news here is the fact that this has to result in a huge spread in aperture required for resolution exponentially depending on faintness. At this stage, it was clear this required additional experiments to get an idea how this really works, but this was scheduled as future task and I proceeded with a rather crude first concept.
- Light pollution or NEML: It is obvious that the effect of light pollution is negligible for bright stars but has an effect on the resolution of fainter stars. The above assumed rA5 form of NEML influence thus needs some modification depending on the magnitude of the secondary
- Influence of M1: Data analysis showed a somewhat counter-intuitive small to no influence of increasing M1 numbers with increasing separation, but it seems obvious that a wide pair of 8mag components is not really harder to resolve than a 6mag pair so I had to insert an additional component into rA3 to represent this effect. First I tried to offload this task completely to the statistical analysis program by using additional parameters but this proved then to be too complicated to be solved numericall. So I had to invent a constant combined with a parameter.

Rule of Thumb Proposing the Size of Aperture Expected to be Sufficient to Resolve Double Stars ...

At this stage I was also in contact with Paul Rodman (software developer of AstroPlanner), who already had Chris Lord's algorithm and also the mentioned Fuzzy Logic Difficulty Index implemented in his software. What I had done so far was, in his eyes, worth being implemented in his software as just another feature supporting double star session planning. This state of the algorithm was frozen as WRAKs RoT Beta 3.1 version. See Table 2 for results. It works rather well for small refractors up to 150mm with some caveats especially for faint wide pairs which means that the problem of telescope magnitude limit was not handled properly. Within the given data set, the correlation coefficient was >0.9 and the standard deviation ~14mm meaning a reasonable spread. As this moment I aimed to get a standard deviation as small as possible and realized only later that some spread is not only unavoidable but even desired to cover the many factors not handled by the algorithm.

While some weaknesses are evident (especially the result for STF2795 with the large CO of 0.35 is obviously off) the overall performance seems to be not bad.

To get a better grip on the TML topic, I eventually started to record the observed telescope magnitude limit at the beginning of each double star observation session usually by looking at Open Clusters or wide multiple stars in the field of view of my planned session. This procedure involved again the use of aperture masks or iris diaphragms to reduce the aperture after locating a faint single star to the degree of non-resolution. I quickly became aware that we have here again the same troublesome situation as with faint double stars; highly unreliable magnitude data. Vmags provided from UCAC4 and URAT1 based on AAVSO APASS were used when available, otherwise UCAC4 and URAT1 fmags were used as estimations. The data set gained by this procedure should then be the basis for a separate analysis of telescope magnitude limit.

To keep an eye on the overall performance of the so far developed RoT algorithm I routinely checked the RoT results by sorting the used apertures by size and comparing them with the RoT results as shown in Figure 1. The use of "switches" to activate the different modules RA1 to RA5 as mentioned above resulted in several jumps indicating room for improvement.

Table 2: RoT Beta 3.1 results for proposed aperture for 50% probability resolution compared with effective measurements for some STF objects

Object	Meas. Aperture	Prop. Aperture	Sep. "	M1	M2	NEML	CO
STF2786	55	61	2.80	7.49	8.20	2.8	0
STF2786	60	61	2.80	7.49	8.20	3.5	0
STF2791	70	74	2.80	8.92	9.28	4.7	0
STF2791	75	77	2.80	8.92	9.28	3.5	0
STF2792	85	81	7.20	9.20	10.60	4.5	0
STF2795	95	110	1.70	9.31	9.66	2.8	0
STF2795	105	110	1.70	9.31	9.66	2.8	0
STF2795	110	113	1.70	9.31	9.66	2	0
STF2795	140	103	1.70	9.31	9.66	2.8	0.35
STF2810	35	44	17.00	8.43	9.04	2.8	0
STF2812	80	95	2.30	9.25	9.70	2.8	0
STF2813	50	62	10.40	9.21	9.72	2.8	0
STF 282	62	65	6.70	9.47	9.51	2.65	0
STF2822	65	73	1.80	4.80	6.20	4.5	0
STF2822	90	73	1.80	4.80	6.20	3	0

I also proceeded to record additional limit aperture observations with the benefit of a running counter-check of the mentioned beta version – this made me also far more aware of potential magnitude data issues as any case of significant deviation from the suggested value required some research to explain the difference and in most cases I ended up with suspect magnitudes but without much luck in finding reliable data in other catalogs.

Eventually I got aware of the possibility to do differential photometry bases on images taken with remote telescopes using the AAVSO VPhot online utility –

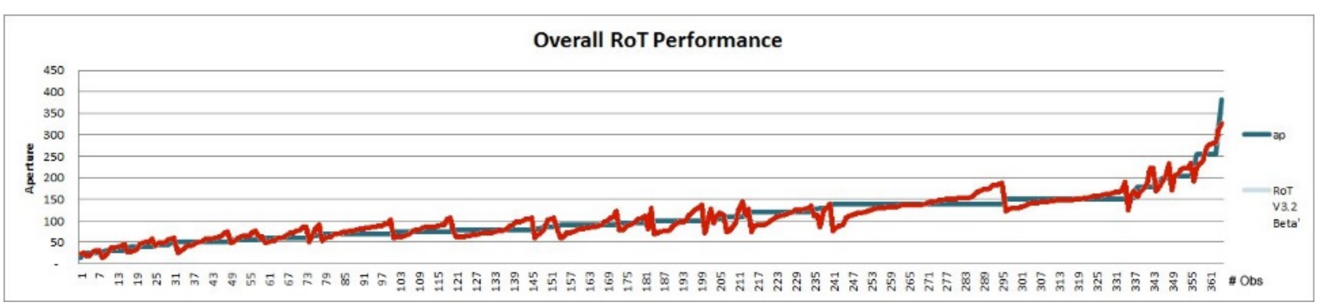

Figure 1. Comparison of measured limit aperture with RoT V3.2 Beta results

first experiments with Landolt reference stars (Knapp 2015) yielded quite satisfying results so I started to keep a separate record of suspect objects to be counter-checked with own images. Later on I switched to easier to use locally running photometry programs like Astrometrica. Most such measurements were then also included in JDSO reports to feed the WDS catalog with precise magnitude measurements but main target was to have within a reasonable error range correct magnitude data available for my limit observation data set. To keep control, I marked such data with my own measurement instead of WDS values in the spreadsheet with a green background for the data fields.

To counter-check the assumed effects of CO I did some experiments with different sizes of central obstruction. These experiments were done with refractors by attaching a spider over the aperture and applying solid disks of different size in the center of the aperture again combined with an iris diaphragm for complete flexibility regarding CO size in relation to aperture (see image 30 in the Appendix). For the owner of a high quality APO already a small speck of dust on the front lens is reason enough to be uneasy so this procedure required some time to get used to it and the most amazing thing was how little to no effect a moderate size CO has on the performance of a refractor for resolving double stars. The usual procedure was as follows: Locating and resolving of a double star, reducing the aperture to the border of non-resolution and then applying increasing sizes of CO to the degree of a noticeable degradation of resolution power of the telescope. The overall results of several experimental session are the following:

- Very small CO of 0.1 or less has no noticeable effect on the resolution power of a telescope.
- Small CO of around 0.15 seems to offer some small resolution advantage for close doubles as expected from diffraction theory but has no effect for wide pairs.
- Moderate CO of up to 0.25 seems again to have little to no noticeable effect.
- Large CO of ~0.3 leads to an increasing negative effect for resolving close doubles but has no effect on wide doubles.
- Very large CO of >0.35 has a heavy impact on resolving close pairs but as to expect again no effect on wide doubles.

I understand that all owners of reflectors might be offended by such a statement on the performance of their equipment, but this is simply what I have noticed during my CO experiments. By sampling such observations and adding them to the limit observations data set, I tried to fill up the data set with enough data points for useful statistical analysis.

At this stage, with the help of XLSTAT, I also did a relevance analysis for the parameters I considered so far as essential for estimating the required aperture for resolution of a double and came up with the not very surprising result that all considered parameters are relevant with a p-value of less than 0.01 (meaning very high probability for being statistically relevant) with the exception of the magnitude of the primary anyway already given by Delta_M and the magnitude of the secondary. So a final RoT version could do without this parameter.

In the second half of 2016 the process of sampling limit aperture observations came to a grinding halt due to the ongoing bad weather with only a few nights allowing for observation sessions at all but even then with mostly rather bad seeing conditions making especially limit observations with apertures larger than 150mm more or less impossible. As I had up to this point of time already over 1000 observations recorded I decided to call it done although I would very much have preferred to add more observations especially in the 150 to 200mm range and also more with different CO sizes.

In the next step I cleaned up the existing data set by checking carefully for errors including eliminating records with obvious absurd results and by trying to get my own measurements for all objects still marked for suspect data.

Starting New Again

I then decided to start RoT model development completely new from scratch using the experience I gained so far from working on the earlier RoT algorithm versions to do it better with a now much larger data set.

General considerations:

- The basic idea is that the minimum required aperture (in the sense of a statistical average, does not mean an effective "limit") is given by the larger value given either by the Dawes criterion for the most simple case of equal brightness or by the telescope magnitude limit corresponding with the faintness of the secondary. This means for example that for a close double with a reasonable bright secondary the required aperture for resolution cannot be much smaller as 116/separation and that for a double with a faint secondary the required aperture cannot be much smaller than required by the telescope magnitude limit estimated as 2.7+5*Log10 (Ap_mm).
- Both starting points are influenced by the size of

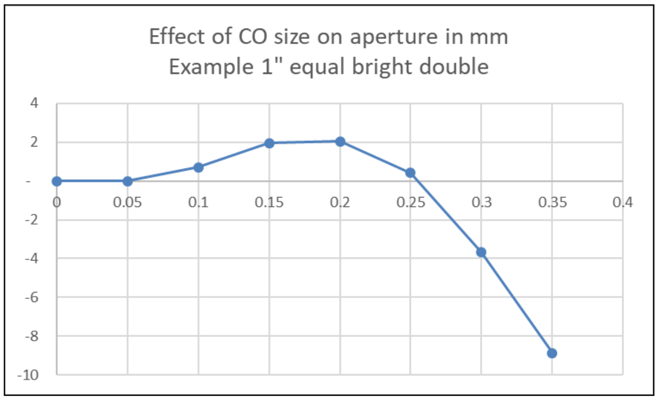

Figure 2. Assumed CO size function based on experiments by modifying the effect of CO on the diffraction pattern

central obstruction – Dawes criterion by change in the diffraction pattern and TML by changing the unobstructed surface.

- For the influence of CO on the Dawes criterion we can combine the results of my experiments with different CO sizes and the changes of the diffraction pattern. The reduction of the central disk (see Appendix table 4) allows for easier resolution of bright close pairs so we can simply use diffraction theory for calculating this effect accordingly – this gives then for example for an 1" equal bright double an aperture of 116mm with zero CO and 108mm with CO of 0.25 (see Appendix table 5). Next comes then the effects of CO noticed by experiments meaning that this effect might if only to some degree come true with a CO size of ~0.15, smaller CO has no effect at all and larger CO up to 0.25 makes this advantage disappear and even larger has increasing negative effects – at least when combined with less than perfect seeing. I am fully aware that regular users of reflector telescopes might have serious doubts on such an effect of central obstruction – selecting then as input for the RoT algorithm a CO size reflecting the own experiences might be an acceptable compromise here.
- This gives in total the following modified Dawes criterion: 116/Sep-Rounded(116/Sep-116/1.2213*(0.0950502775050452+(1.12627632206642)/((1+(CO/0.302756091410027)^2.26536793426585) ^2.26536793426585))/Sep;0)*(-0.124502804842503+15.5919411863431*CO-79.952641306428*CO^2+46.497636868053*CO^3+180.046972257086*CO^4-96.0995272278428*CO^5-312.155425754896*CO^6+252.108685457266*CO^7).
- With Sep = separation in arcseconds and CO = size of central obstruction from zero to less than one. The first part of the formula corresponds with the change of the size of the central star disk according to diffraction theory. The second part is a statistical approximation to the assumed effect of CO size on resolution as visualized in Figure 2.
- Figure 3 is a visualization of this function.
- For the influence of CO size on TML we have simply to calculate the corresponding aperture size for the unobstructed surface. Additionally we have to consider that a multitude of factors like light pollution, transparency site's elevation, season (winter or summer) and elevation of the star observed influence the resolution of stars by dimming the "absolute" brightness of the star – with double star observation we are on the lucky side of this effect as while the influence on the naked eye magnitude limit seems dramatic this means when observed through a telescope (else fair seeing given) only a small loss in magnitude. If we observe a bright 6th magnitude pair and lose due to overall extinction for example 0.3mag then we have a 6.3mag pair and this has certainly little to no effect on the required aperture for resolution. But the effect is very well a strong one when we come to fainter stars where the same factors may increase the extinction up to 1 mag or even more. This means we have to reconsider the topic of telescope magnitude limit again and also the possible influence on the magnitudes of both components before doing statistical analysis of the limit observation data set. Phillip Creed tried to catch this effects in his spreadsheet (Creed 2007) with a bit less complexity than Schaefer but the attempt to building up TML with NEML as starting point is despite on first look plausible results doomed to fail because as the com-

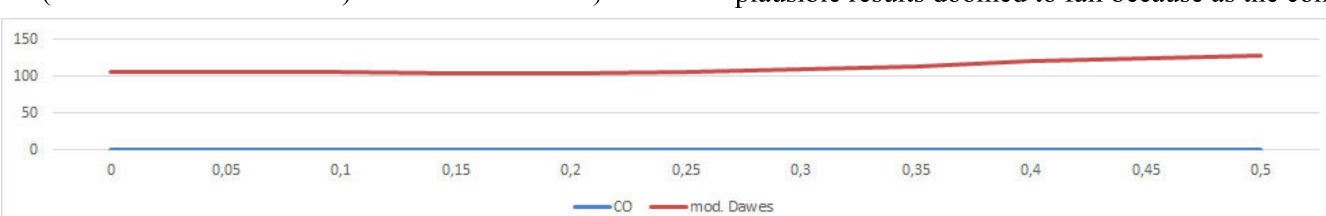

Image 3: Visualization of the modified Dawes function

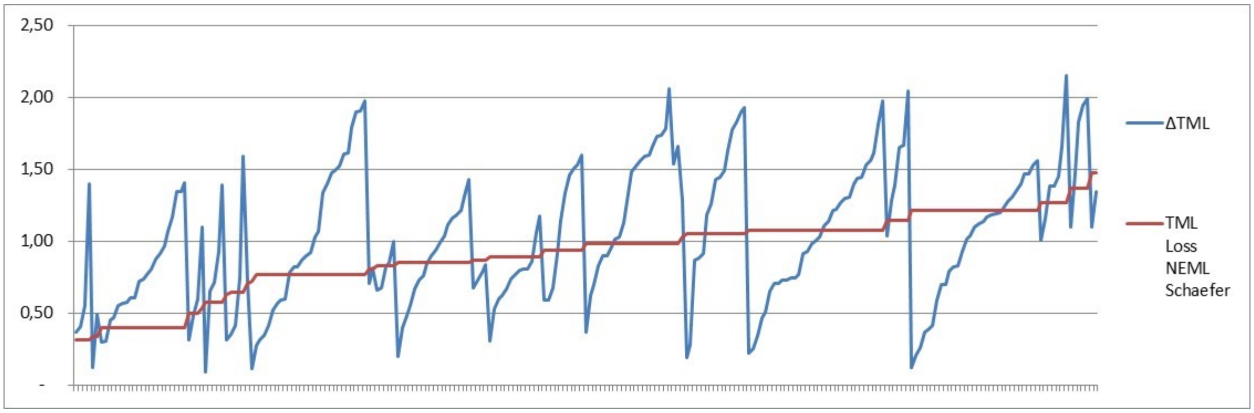

Figure 4. Comparison of measured TML loss with Schaefer's model

parison with my data set of ~250 TML observation shows clearly that the effect of NEML on resolution of faint objects is certainly not linear but logarithmic (Chris Lord's Excel version of Schaefer's model is based on the same erroneous assumption - Lord 2008). An additional issue with NEML determination is the fact that we have a limited choice of stars in the zenith field of view at any given moment and this choice might also be marred by color issues – as a matter of fact the mentioned data set did not show the expected behavior that the measured delta to the calculated TML depends in a clear way on NEML as the spread of the observed data points is erratic to a degree that statistical analysis seemed useless. I resorted then to using the part of Schaefer's model dealing with the influence of NEML on TML by keeping all parameters but NEML unchanged and checking the delta to TML with NEML 6.5mag. The comparison of observed delta to 2.7+5*Log10(mm) to this model sorted by ΔNEML (=6.5-NEML) is given in Figure 4.

The spread is with a standard deviation of 0.45mag obviously huge (interestingly Schaefer 1990 also reports a typical model error of 0.5mag) and can in my opinion only be explained by systematic flaws in the determination of NEML and TML due to a multitude of factors including the despite the APASS efforts questionable reliability of given mag values for faint stars and color issues – yet the impression remains that the Schaefer model delivers here a good estimation for the average NEML effect on TML. What is not to be found neither in my own data nor in Schaefer's model is the effect on aperture size I expected here. The larger the aperture, the larger the TML loss should be because the difference in terms of absolute brightness between mag 14 to 13 is certainly much smaller than for example 11 to 10 but this assumption did not work out.

A subset of the Limit Aperture Observation Data Set consists of ~80 very wide pairs with the magnitude of the faint companion being the limiting factor for the resolution. The comparison of observed limit aperture with the calculated limit aperture for the given magnitude of the secondary combined with NEML looks for this subset as shown in Figure 5.

This looks with the exception of a few very obvious

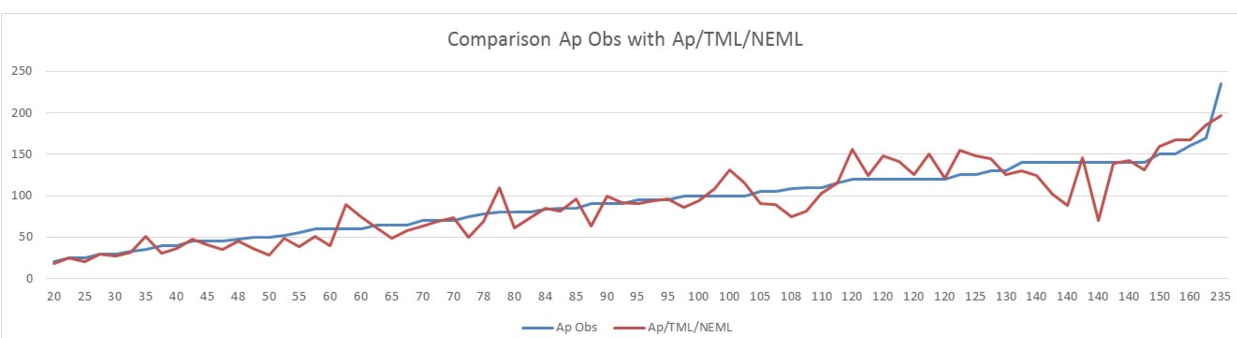

Figure 5: Comparison of the measured limit aperture for very wide pairs with the calculated aperture according to the modified TML function

Rule of Thumb Proposing the Size of Aperture Expected to be Sufficient to Resolve Double Stars ...

outliers especially in the 140mm aperture range quite good and led in consequence to the elimination of two suspect and probably erroneous records from the overall data set as the counter-check showed in one case a meanwhile changed magnitude for the secondary and a potential mix up of components in the second case. Other obvious outliers as for example ES 1906 with a very faint secondary in the 13.6mag range resolved with 140mm aperture and 3.3mag NEML remain unexplained – such a result seems impossible besides may be for blue-white components but both are according to own measurements with V- and I-filter rather in the orange spectrum.

The final modified TML function is the following:

SQRT(((10^((M2+0.0149136546170395+0.124667306072993*(6.5-NEML)^1.63506511158234 -2.7)/5) /2)^2*PI+((10^((M2+0.0149136546170395+0.124667306072993*(6.5-NEML)^1.63506511158234 -2.7)/5) *CO)/2)^2*PI)/PI)*2

with M2 = magnitude of the fainter star of the pair, NEML = Naked Eye Magnitude Limit, CO = size of CO and PI = value for Pi.

Graphs of this function are shown in Figures 6-8.

By considering both modified starting points (means either modified Dawes or modified TML) as the lowest reasonable value for the required aperture size we get the following results:

- First the modified Dawes criterion as illustrated in in Figure 9. With a few exceptions all measured aperture sizes are larger than the modified Dawes criterion, indicating a few over-performing observations.
- Second the modified TML is illustrated in Figure

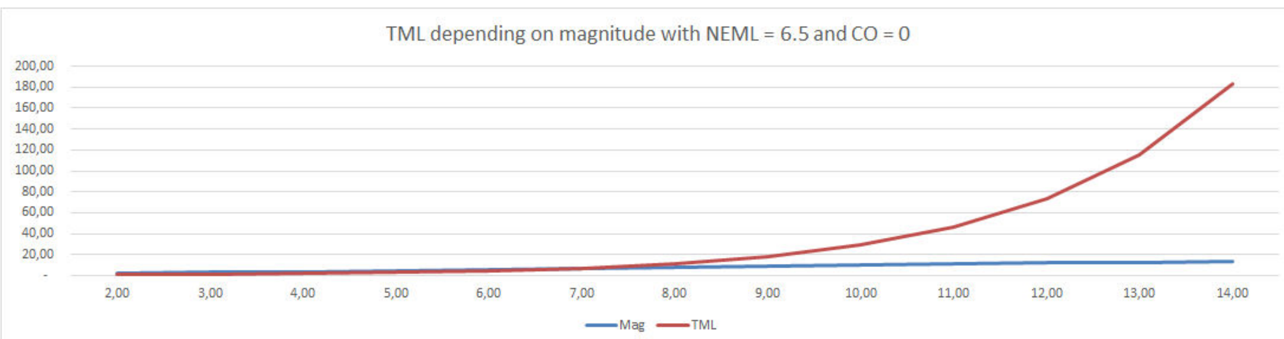

Figure 6. Calculated aperture according to the modified TML function for increasing faintness of stars.

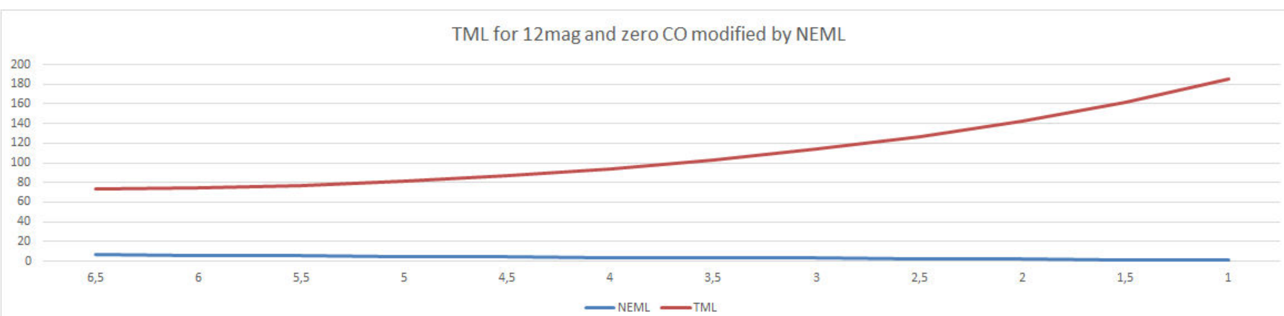

Figure 7. Calculated aperture according to the modified TML function for increasing light pollution.

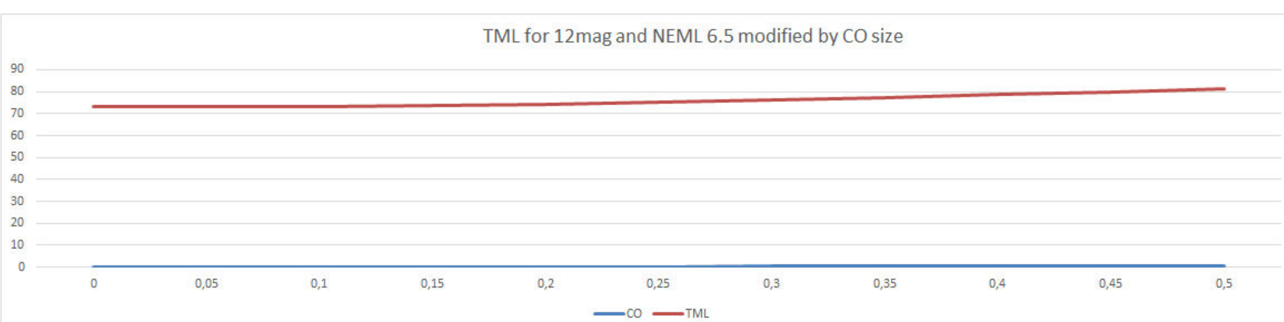

Figure 8. Calculated aperture according to the modified TML function for increasing central obstruction.

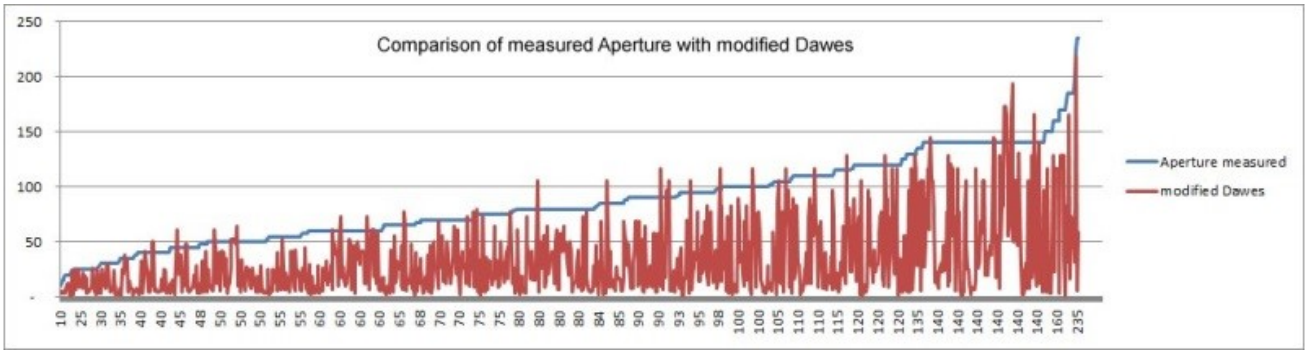

Figure 9. Comparison of measured limit aperture with the modified Dawes function.

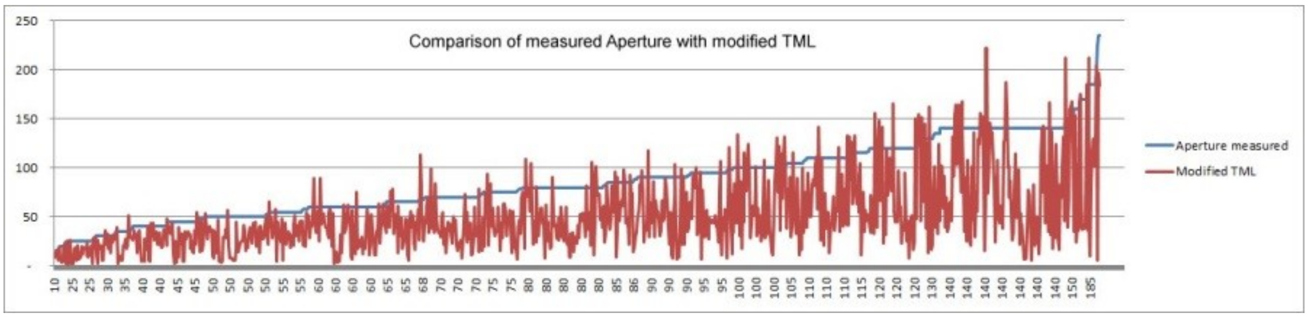

Figure 10. Comparison of measured limit aperture with the modified TML function.

10.

Regarding comparison measured apertures with modified TML we get a similar picture – even if a few more observations "beat" for whatever reason the calculated TML.

So while both criteria do not work perfectly in the sense of covering 100% of all measurements, they seem to be an acceptable compromise as starting points for developing a RoT algorithm starting with

$$pA = (pr_1)(D_{\text{mod}})^{pr_2} + (pr_3)(T_{\text{mod}})^{pr_4} + \ldots$$

where pA = proposed Aperture, D_{mod} = modified Dawes criterion, T_{mod} = modified TML and pr_1 to pr_4 as parameters with a value to determine by statistical analysis. The structure of $pr_1 * X^{pr2}$ allows for linear and nonlinear effects of a given variable assuming a consistent influence of this kind of the variable but does not cover any possible counter-effects with changing values.

We can proceed now with the doubtless most important parameter for unequal bright doubles: ΔM in relation to the separation means exponential increasing effect with smaller separations. I experimented with several models on how to do this and got the most promising results with

$$p_A = (pr_1)(D_{\text{mod}})^{pr_2} + (pr_3)(T_{\text{mod}})^{pr_4} + (pr_5)\left(\frac{\Delta M}{Sep}\right)^{pr_6} + pr_7$$

with all parameters as above plus ΔM for delta_M and Sep for separation and pr5 to pr7 as parameters with a value to determine by statistical analysis with pr7 to be seen as residual over all observations. This resulted in a model with a correlation of 0.9222 and a standard deviation of 14.28mm regarding aperture size – may be not perfect but not bad. See Figure 11.

To avoid potential numerical problems with delta_M less than 1, I set delta_M to 1 for all objects with a value less than that and tried to get a better result with modifying the model and ended so far up with the following approach giving delta_M a double appearance:

$$pA = (pr_1)(D_{\text{mod}})^{pr_2} + (pr_3)(T_{\text{mod}})^{pr_4} + (pr_5)\frac{(\Delta M_{\text{mod}})^{pr_6}}{(Sep)^{pr_7}}$$
$$+ (pr_8)(\Delta M_{\text{mod}})^{pr_9} + pr_{10}$$

This brought, then, a tiny enhancement of the results with correlation of 0.9234 and standard deviation of 14.17. See Figure 12.

The parameter results I got are:

- pr1 = 28.2004379647114

Rule of Thumb Proposing the Size of Aperture Expected to be Sufficient to Resolve Double Stars ...

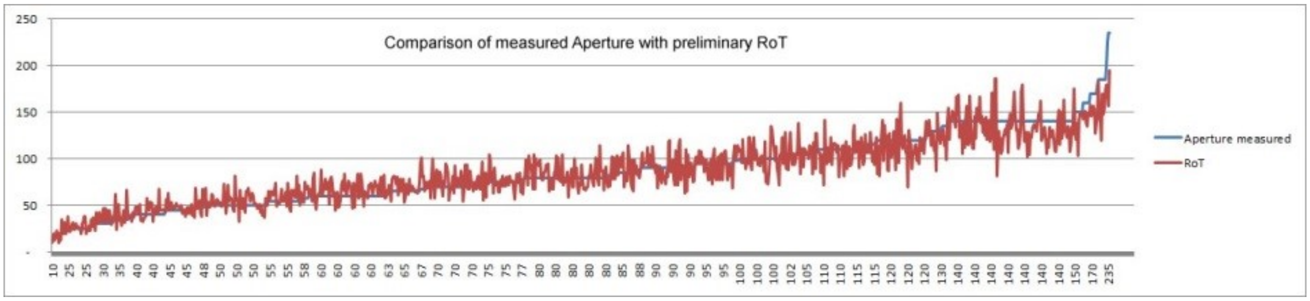

Figure 11. Comparison of measured limit aperture with the preliminary RoT function..

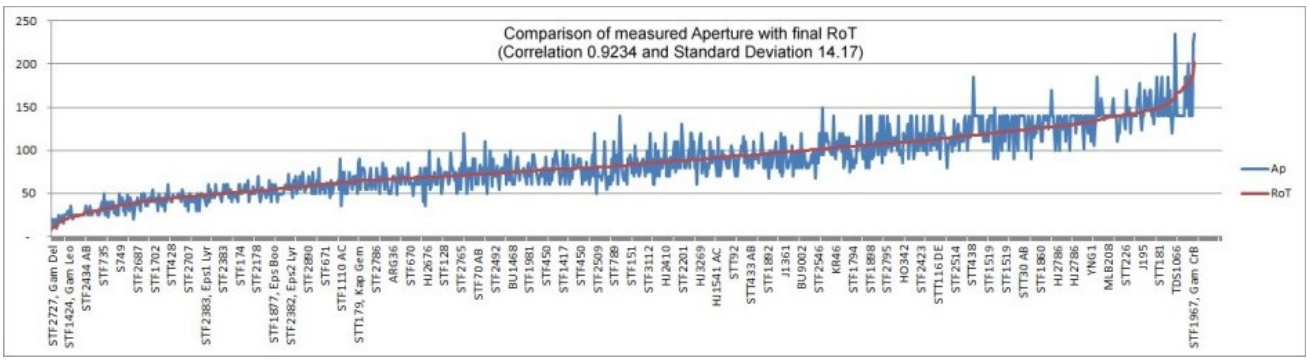

Figure 12. Comparison of measured limit aperture with the final RoT function..

- pr2 = 0.54341406881422
- pr3 = 7.93181801181747
- pr4 = 0.57008922996566
- pr5 = -279.748706397389
- pr6 = -0.076600150962929
- pr7 = 0.461363131302114
- pr8 = 8.14981519358482
- pr9 = 0.468237554468765
- pr10 = -26.8211959485956

The visual comparison does not show significant changes but as stated the statistical "curve adapting" is slightly better. May be it would be possible to get better results with a completely different model approach but I decided to keep it at that – with the given spread for the same or very similar parameter values in the data set no statistical analysis can yield much better results. The values of the parameters need no interpretation as they are simply the result of statistical curve adapting but the value for pr10 suggests that there might be input values resulting in a negative aperture size what is obviously nonsense. But we can conclude from experience that any result less than 20mm aperture means that his has to be an object for a binocular and not a telescope how small ever.

Some huge spikes in the graphical comparison of used aperture compared to proposed RoT values indicated outliers with the most significant counter-checked in the data set for plausibility.

Simply sorting the data set offers already some interesting insights:

- Sort by object shows for many objects rather consistent results but for many objects also the spread of measured limit apertures in different sessions despite seemingly similar conditions and demonstrates that any RoT has to deal with a significant spread in measured limit apertures – but within realistic range there is certainly no reason to consider this as outlier but simply an effect of different seeing conditions even if for example STF2514 with two observations with a difference of 50mm in aperture with only a minor difference in NEML seems quite suspect.
- Sort by separation offers also interesting insights with similar results with comparable parameters but often also significant scatter. And in some cases this offers also hints for outliers if the delta between the results seems beyond a reasonable spread suggesting a data error.
- Sort by magnitude of the secondary indicates outliers by seemingly absurd combinations of used aperture to magnitude – for example ES1906 with 140mm aperture for a 13.58mag companion with 3.3mag NEML. This suggests either a huge data error or a blue-white star making resolution far easier as the magnitude suggests but counter-checks

showed that neither is the case.

Just for testing the influence of potential outliers on the final result I eliminated some of the assumed outliers and run again the same statistical analysis to show a potential negative influence and got (not really a surprise) a very similar result – this means that a few odd data records are within this large number of observations of little consequence but a slightly better correlation and a somewhat smaller standard deviation. So the mentioned outliers have only been eliminated for this experiment but are still included in the full data set as the benefit from trying to eliminate outliers seems rather negligible – yet all these objects are subject to counter-checks with own measurements not only for the basic parameters but also regarding spectral class on the lookout for explanations.

One very interesting insight when trying several attempts of statistical analyses, was the influence of different sequences of model parameters or data records on the resulting parameter values gained with XLSTAT but all with rather similar correlation and standard deviation. The XLSTAT non-linear regression data analysis also begins with a random start value. This is another reason why the presented "final" solution for the parameter values is only one of a class of very similar ones. This means also that another model structure than the one chosen here might even provide better results in terms of correlation and standard deviation.

Analysis of the RoT Behavior with Different Values for the Variables

I checked the behavior of the proposed "final" RoT result for different scenarios.

Figure 13 shows the expected exponential behavior, but is rather optimistic when compared with the standard Dawes criterion. This is a side effect of projection beyond 0.6" separation (smallest separation included in the data set).

Figure 14 shows the expected slightly exponential behavior for increasing faintness of the companion (compared with the constant value of the Dawes criterion).

As expected the influence of light pollution is while given rather small as shown in Figure 15. Perfect black sky gives Rot results slightly smaller than the Dawes criterion with "break even" at the NEML value of ~3.3mag.

Figure 16 shows the expected slightly exponential results for increasing faintness of pairs with unchanged ΔM (compared with the constant value of the Dawes

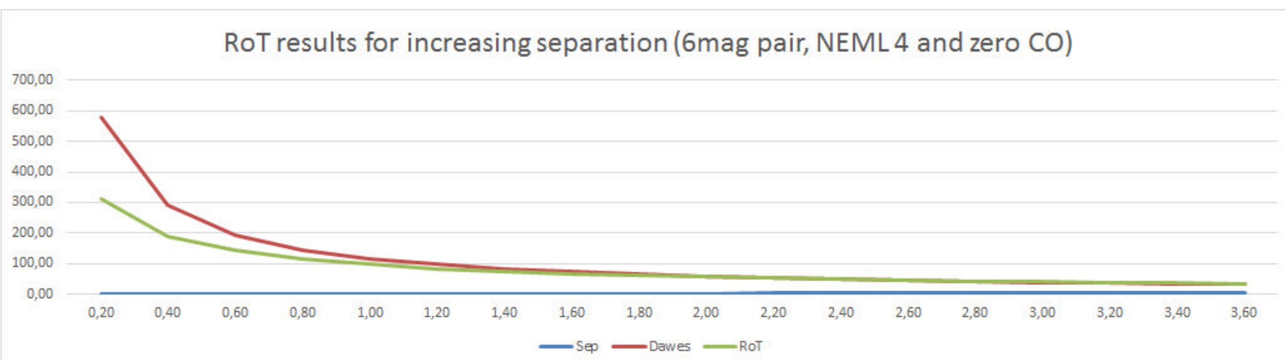

Figure 13. RoT results for increasing separation.

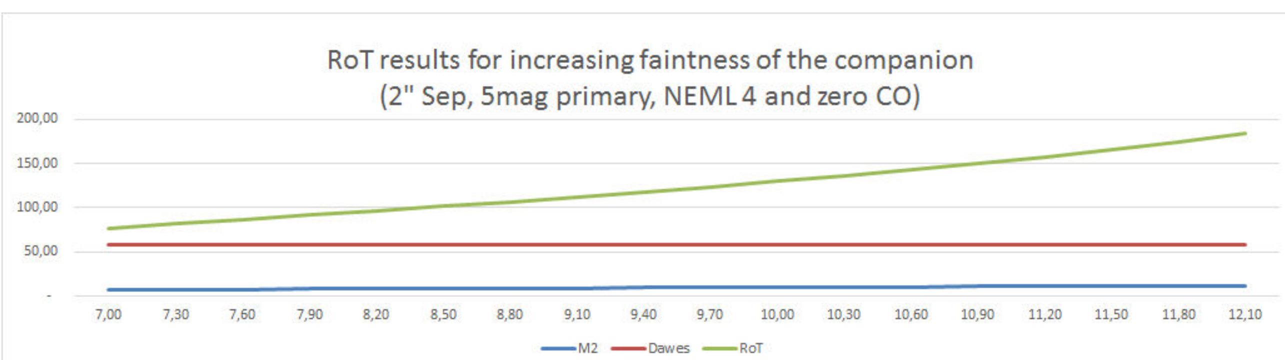

Figure 14. RoT results for increasing faintness of the companion.

Rule of Thumb Proposing the Size of Aperture Expected to be Sufficient to Resolve Double Stars ...

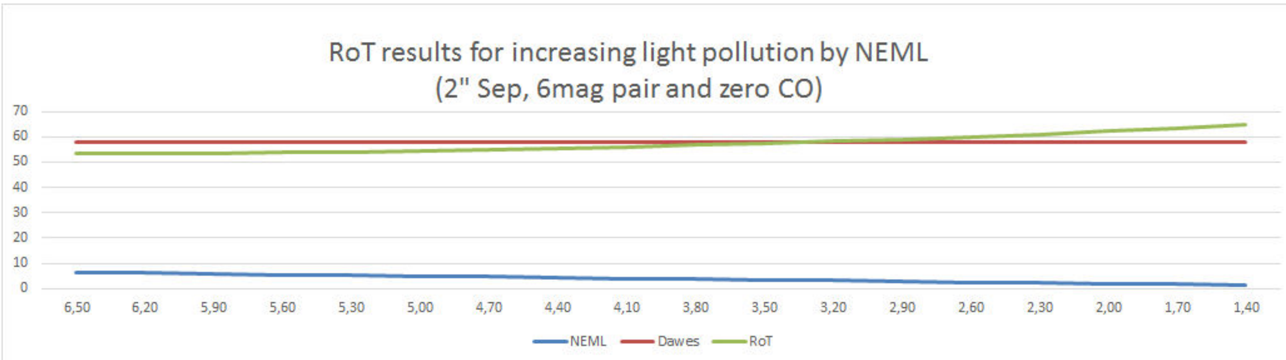

Figure 15. RoT results for increasing light pollution..

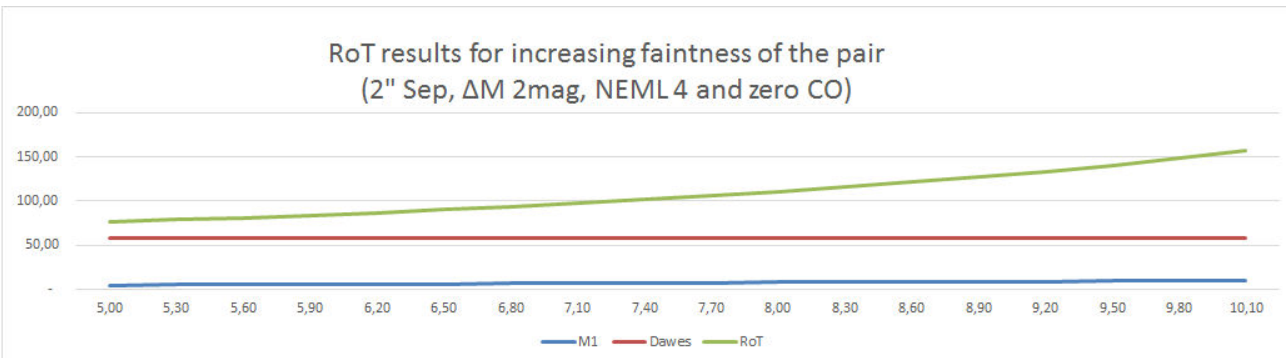

Figure 16. RoT results for increasing faintness of both components with given ΔM..

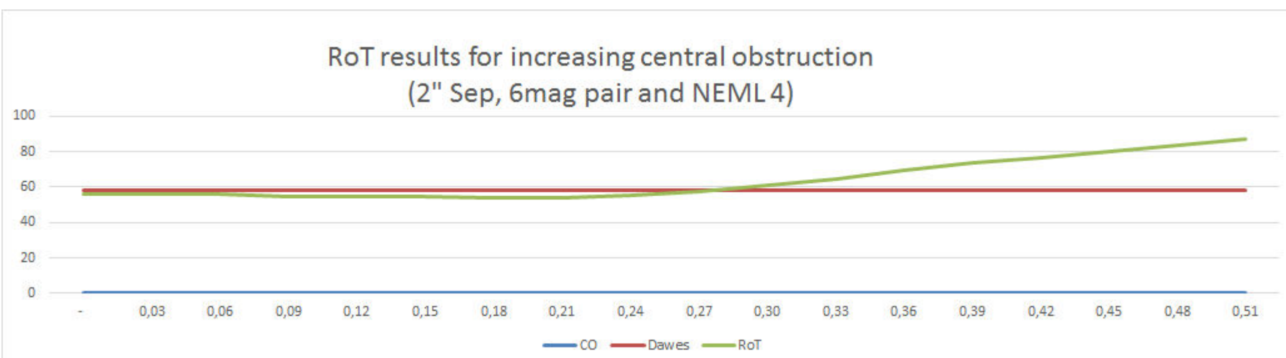

Figure 17. RoT results for increasing central obstruction.

criterion).

Figure 17 shows that, as expected, there is a small advantage compared to Dawes with a "break even" around CO 0.25 and getting worse with CO significant larger than 0.3.

Overall there is the impression of a graceful behavior or the RoT algorithm with changing values of the different variables used.

Comparison of RoT Results with Different Variable Values in the Full Data Set

In Figures 18-26, I checked the behavior of the proposed "final" RoT result for different values of variables in the data set:

The objects listed in Figure 27 will be subject of further investigation.

Overall 36.69% of the objects in the data set have a difference between measured aperture and RoT result of 5mm or less, 35.75% between 6 and 14mm, 21.60% between 15 and 28mm and only 5.96% over 28mm. See Figure 28.

Figure 29 shows the distribution of the used apertures in relation to the RoT results showing not only

(Text continues on page 116)

Rule of Thumb Proposing the Size of Aperture Expected to be Sufficient to Resolve Double Stars ...

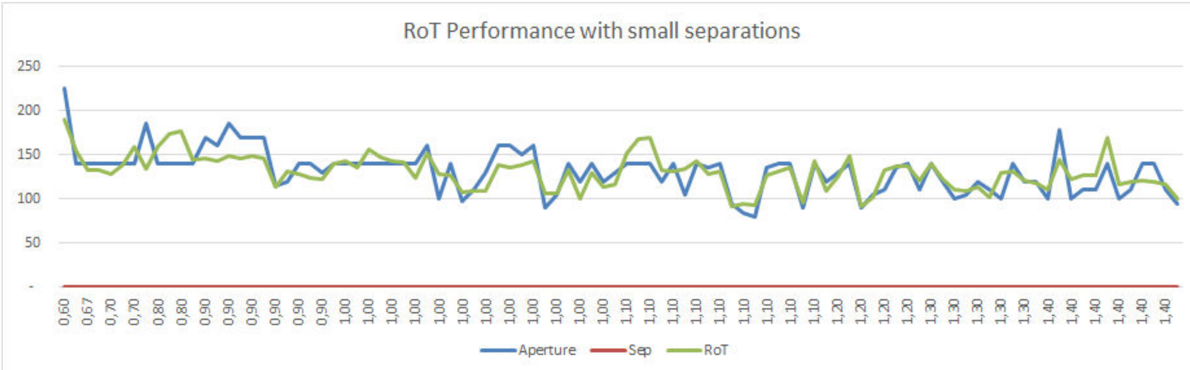

Figure 18. Comparison of RoT results with measured limit aperture for doubles with small separation

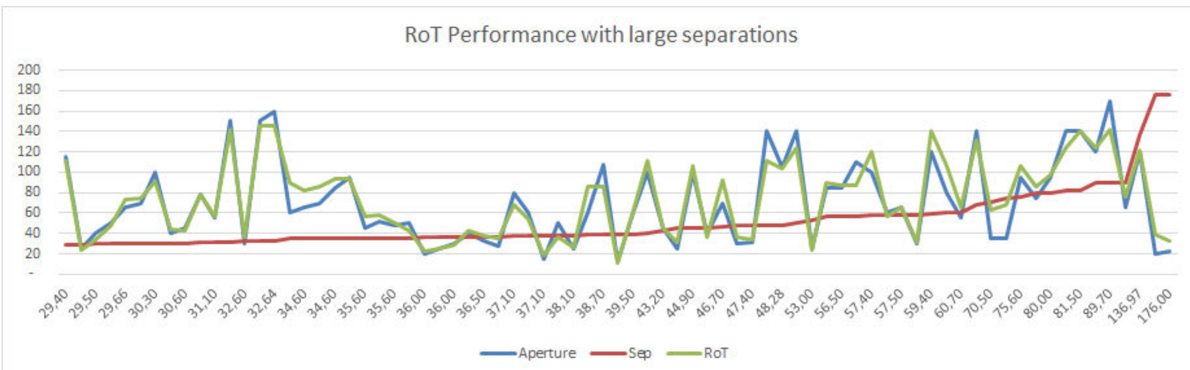

Figure 19. Comparison of RoT results with measured limit aperture for doubles with large separation

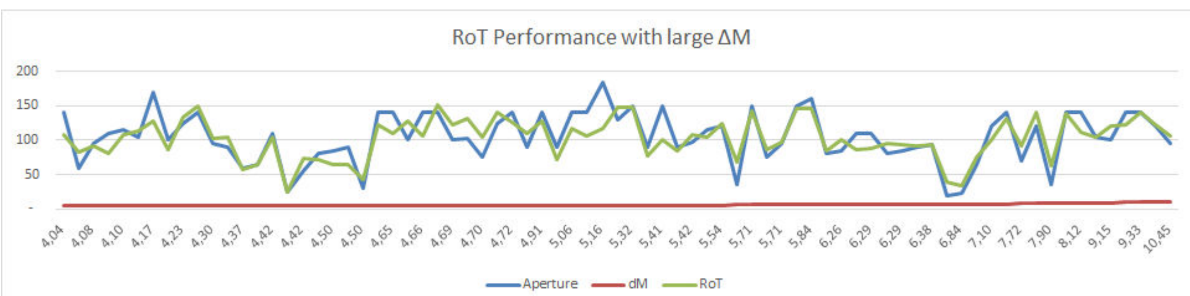

Figure20. Comparison of RoT results with measured limit aperture for doubles with large ΔM

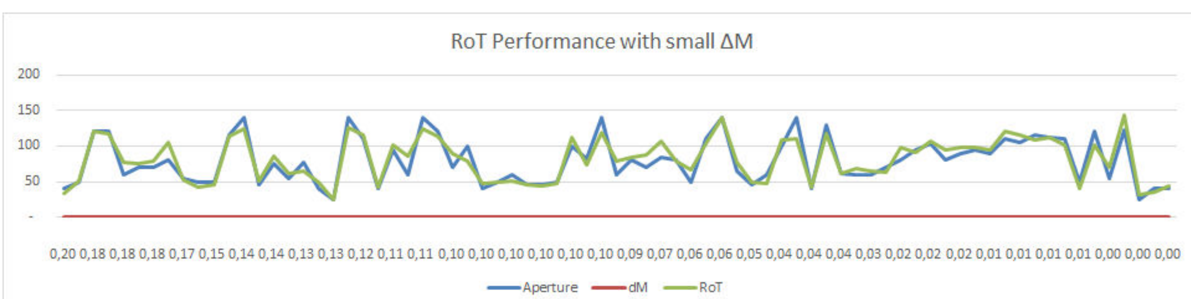

Figure21. Comparison of RoT results with measured limit aperture for doubles with small ΔM

Rule of Thumb Proposing the Size of Aperture Expected to be Sufficient to Resolve Double Stars ...

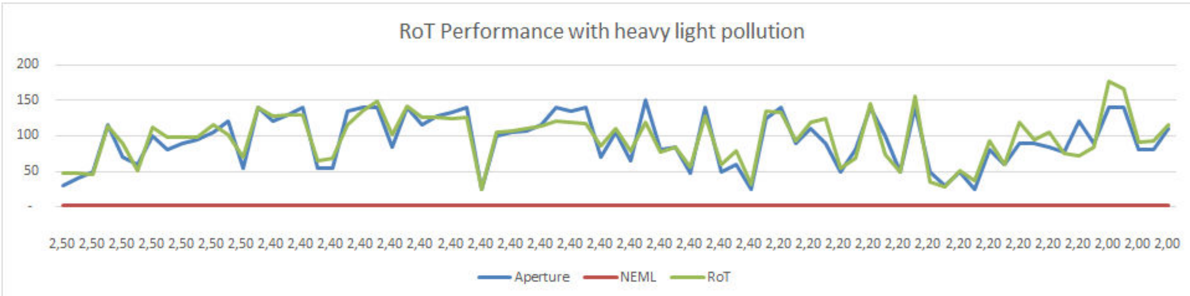

Figure22. Comparison of RoT results with measured limit aperture with heavy light pollution

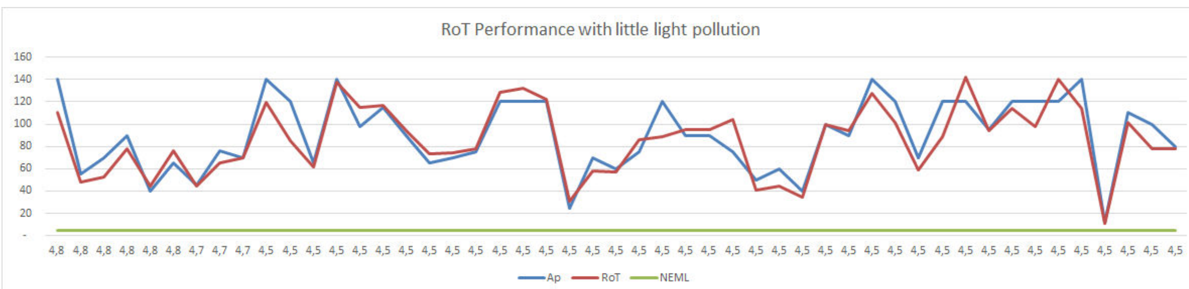

Figure23. Comparison of RoT results with measured limit aperture with little light pollution

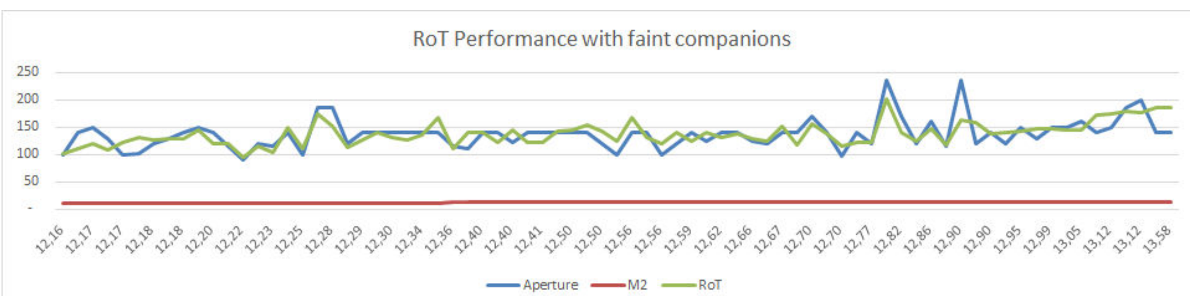

Figure24. Comparison of RoT results with measured limit aperture for doubles with faint companions

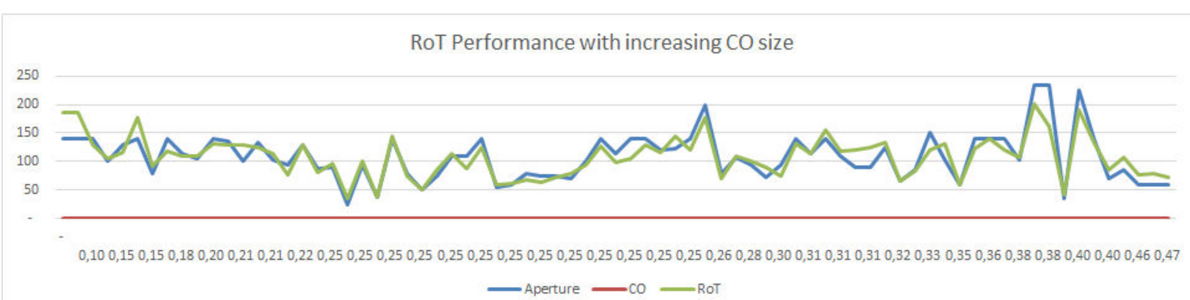

Figure25. Comparison of RoT results with measured limit aperture with increasing CO size

Rule of Thumb Proposing the Size of Aperture Expected to be Sufficient to Resolve Double Stars ...

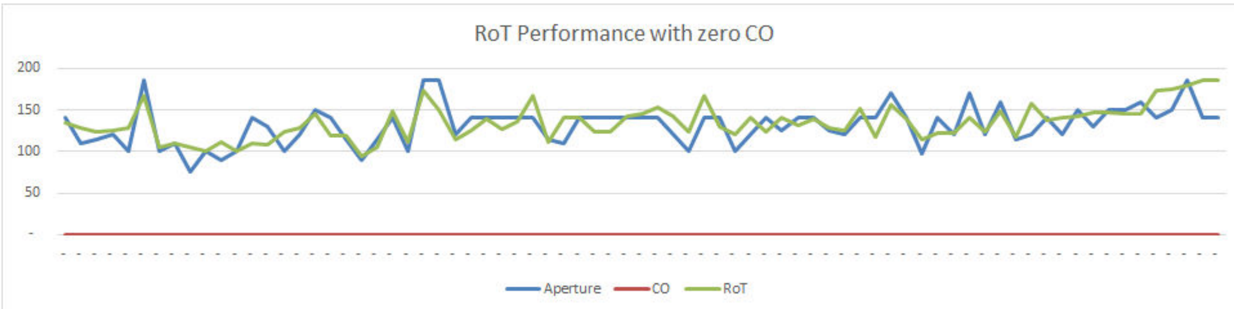

Figure26. Comparison of RoT results with measured limit aperture with zero CO

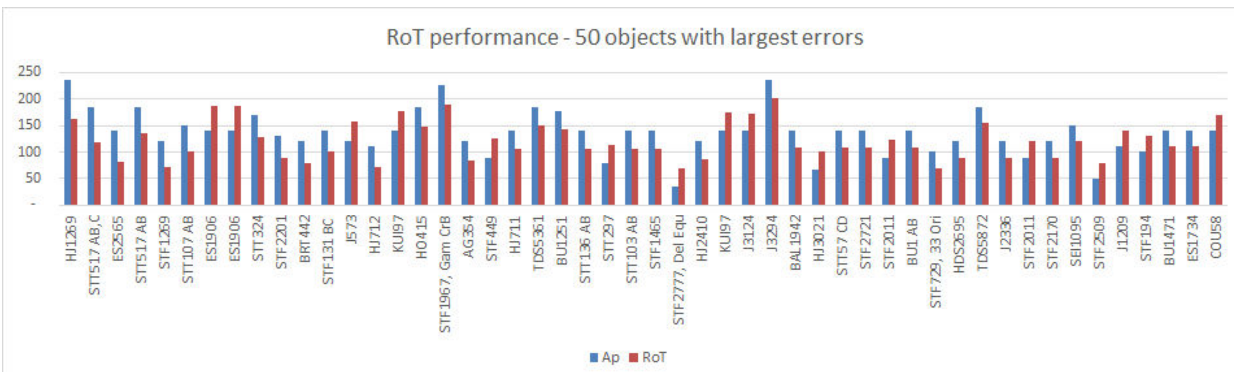

Figure27. Objects with largest difference between RoT results and measured limit aperture

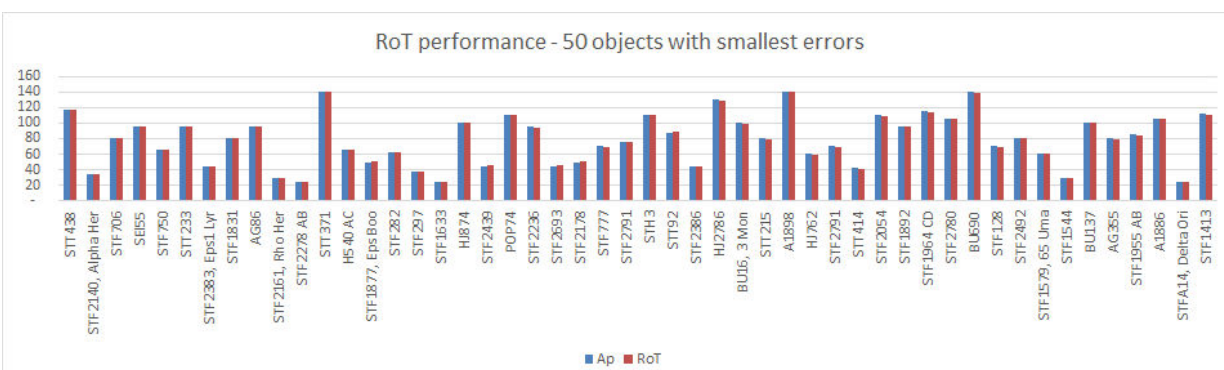

Figure28. Objects with smallest difference between RoT results and measured limit aperture

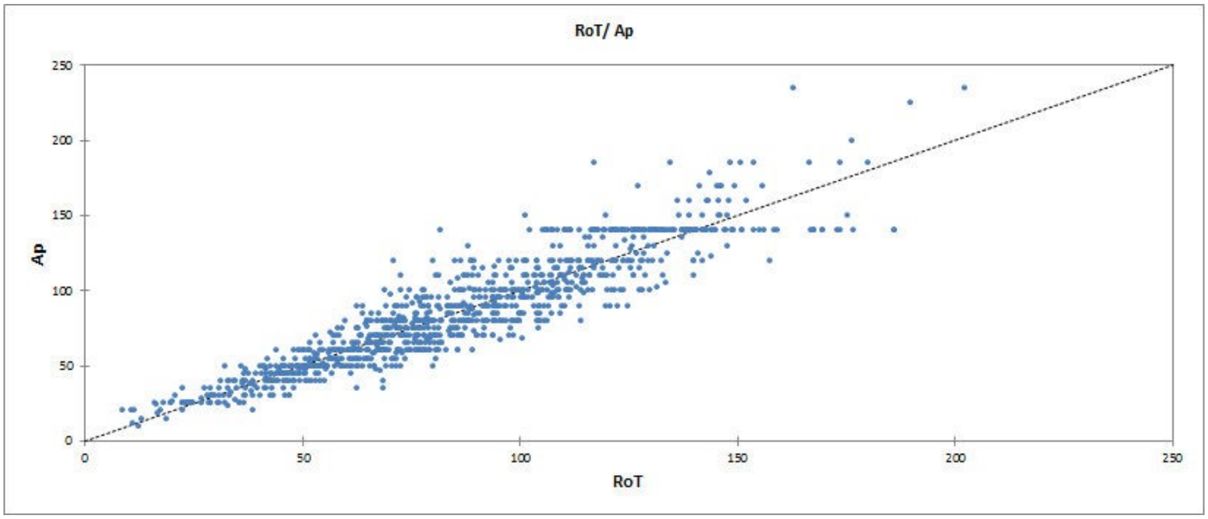

Figure 29. Distribution of RoT results compared with measured limit aperture

(Continued from page 112)

that the number of observations with apertures larger than 150mm is rather small but especially that the number of observation with 140mm is not only very large but shows also a noticeable huge spread. This is obviously a side effect of the predominant use of my favorite 140mm refractor telescope with a tendency to "just resolved" results including potentially false positives, lucky hits or especially good seeing conditions – these are the usual problems with fixed aperture limit observations.

Interpretation of the RoT results

So what does a given RoT value mean for the visual observer – let's take for example KUI 66 (15 Boo) with a proposed aperture of ~150mm. At first look this might be interpreted that the probability for resolution with a 150mm refractor with else given fair conditions is 50% as we know from our statistics that half of the observations in our data set are below the RoT value and the other half above. This would mean that if we observe KUI 66 ten times we could expect resolution in five cases.

But this is certainly not the case as the given RoT value only means that the limit aperture sufficient for resolution is with a 50% chance below the RoT value but with the same probability also above – so the RoT 50% expectation is realistic for a large number of objects but not for the single object. For a single object this might mean that the resolution success rate with the proposed aperture is with average fair seeing conditions 100% or zero if we have bad luck. But then: For a single object we have to consider the statistical spread of ~14mm standard deviation meaning that we have a probability of 67% that the required aperture for resolution lies in the range RoT +/-14mm (assuming normal distribution) – in our example for KUI 66 this means 136 to 164mm aperture. At first look this information seems not as useful as we would have expected but the good news here is that this range might very well cover the spread we experience in the reality of visual observ-

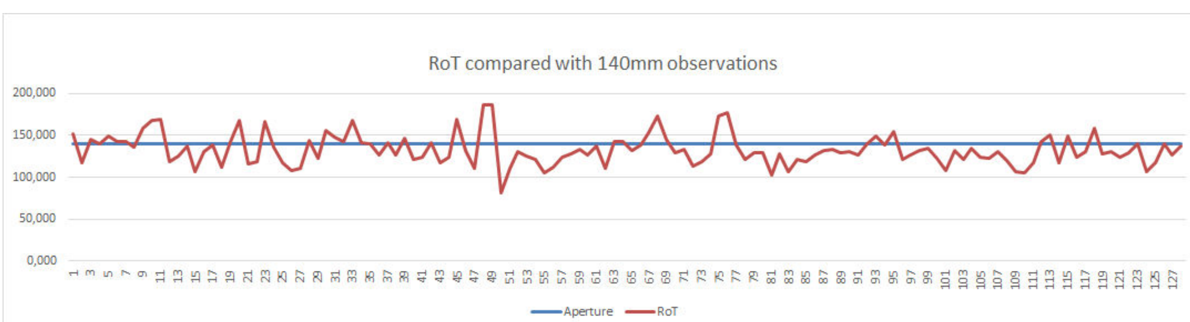

Figure 30. Comparison of RoT results for objects with measured 140mm limit aperture

ing sessions and that we can resolve a given double with the given RoT value under good conditions even if the "real" average aperture required for resolution might be in the upper range of the mentioned spread.

The question remains what we should do with very small RoT results below 25mm – today nobody has a telescope this small (you have to go back to Galileo to find such an instrument) and even most finder scopes are larger than 25mm. So we can only conclude that objects with a very small RoT value are easy targets for even the smallest telescopes available and for objects with RoT values below 20mm we can consider these to be objects suitable for observation with binoculars.

Known Weaknesses

Weaknesses due to parameters not included in the RoT algorithm include:

- Color issues: Double star components in the red spectral range are far harder to resolve than these in the blue-white range. One extreme example in this regard is J 553 with 3.7" separation and 10.59/11.86Vmag (own measurement) – both components are in the blue-white range and resolution is far easier than anticipated. This means that we have to assume yellow light in average with some if little room for minor color issues
- Diffraction pattern issues: The resolution of a faint secondary might be influenced by the position within the diffraction pattern and the brightness of diffraction rings (Treanor 1946) – but to consider such influences in detail would mean to open Pandora's box so I decided to stick to the pure statistical approach although the question of for resolution sufficient brightness of companions sitting on the first diffraction ring seems quite interesting.

Weaknesses of the data set are:

- All observation are done by only one observer – while this means some degree of consistency it means also a high degree of being subjective. Yet I assume average abilities as observer so I hope the resulting RoT parameter values are of general use of double star observers and if there are differences they might prove to be rather systematic and thus easy to evaluate
- Only about 6% of the observations are done with a central obstruction what means that the statistical base for reflectors is rather shaky but as the structure of the CO-module is based on extensive experiments with different sizes of CO we might hope for the best
- Only about 3.5% of the observations are done with limit apertures larger than 140mm what means that the statistical base for apertures larger than 140mm is quite shaky. This means that all RoT results for apertures larger than that have rather the character of projections supported by a few data points
- Observations were made on only two locations with Naked Eye Magnitude Limit from 2 to 4.8mag and usually not this good seeing. I can remember only very few sessions having excellent seeing and in total average moderate fair seeing can be assumed – this means that in locations with usually excellent seeing the RoT results might be a bit on the conservative side.

Minor issues include:

- Primary is usually considered the brighter component – this is not always the case in the WDS catalog. This is solved for ΔM by calculating as absolute difference and for TML by checking for the fainter component of the pair
- Extreme parameters with huge separation and bright components with little Δ_M might as already mentioned in rare cases result in a negative RoT value (one example is famous double double Eps Lyr with 207" separation of the two doubles with a combined magnitude of 4.6 and 4.7mag yielding a proposed aperture of -6mm). This weakness of the algorithm is simply handled with a cut off – any object with a value below 20mm is declared a binocular object.

Summary

While I am fully aware that "The illusion that one has understood the past feeds the further illusion that one can predict and control the future" (Kahneman 2011, page 204) I am confident that the here presented statistically derived Rule of Thumb algorithm (available for download under http://www.sterngucker.eu/RoT/RoT.xlsx) is of some value as already the preliminary versions were in many of my observation sessions counter-checked with very good results for the aperture range of 60-150mm and with some caveats up to 200mm (above the results are to be considered as pure projections without support of empirical data and thus of limited use) and proofed to be useful for several purposes:

- Session planning: Selecting of objects suitable for a given telescope in the above mentioned range
- Counter-checking of object parameters: If an object is despite fair seeing harder to resolve as indicated then further research is paramount as an explanation is necessary: Wrong parameters (especially magnitudes) or color issues are possible causes.

The same situation is given if resolution is far easier than expected

- Estimation of the magnitude of the secondary: Often the given magnitude for the secondary is known to be questionable while separation and magnitude of the primary are to be considered reliable. To determine the required aperture for resolution (requires aperture masks or iris diaphragm) allows to roughly estimate the magnitude of the secondary

A potential follow up might be the extension of the existing data base with additional observation with different sizes of CO and with limit apertures larger than 140mm. The former seems not this critical as the results for refractors might be of good use as reasonable good indications for reflectors for aperture sizes of less than 200mm. For the latter an extended period of excellent seeing conditions would be necessary and this is something one can only hope for but not plan.

Counter-Check

The ongoing Sissy Haas project (http://www.billboublitz.com/Haas_Project/hbsop_index.html) might offer an opportunity for an independent counter-check of the RoT model presented here. For this purpose the object matrix of this project is given in Table 3 with the RoT results for comparison with the expected future final results of this project (taking into account the 14mm standard deviation of the RoT value plus the fact that the reported “final” values for this project might be a bit on the conservative side).

For all results below 60mm it should be remembered that telescopes with less than 60mm aperture are usually not available so there can be no reports for smaller apertures except for a few with aperture masks or an iris diaphragm. For a good part of these objects my own measurements of the limit aperture are to be found in the full data set and are included for comparison purposes.

Reproducibility

To provide the possibility of at least analytical replication (according to Winey 2014) of the presented RoT model the data set of 1,074 observations on which this model is based is available for download at http://www.sterngucker.eu/RoT/LimitObservations.xlsx.

The contents of the site are as follows:

- Object = Discoverer ID
- Ap = Measured limit aperture for resolution
- Sep = Separation according to WDS at the time of the observation (or own measurement if green background)
- M1 = Magnitude primary according to WDS at the time of the observation (or own measurement if green background)
- M2 = Magnitude companion according to WDS at the time of the observation (or own measurement if green background)
- NEML = Naked Eye Magnitude Limit (in the field of view means including extinction)
- CO = Central Obstruction of the used telescope.

Despite including many objects with my own measurements to eliminate suspect data as far as possible, this does not mean that the data of all other listed objects is considered to be correct. On the contrary, some objects are given with different WDS data depending on the time of observation and several objects considered to be outliers I am intending to measure myself for counter-checks.

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- Washington Double Star Catalog
- iTelescope
- AAVSO VPhot
- AAVSO APASS
- UCAC4 catalog via the University of Heidelberg

Table 3. Calculated RoT results for the Sissy Haas project object matrix (based on the actual data of the objects behind each cell with NEML 3.5 and zero CO)

Sep/dM	dM=1	dM=1.5	dM=2	dM=2.5	dM=3	dM=3.5	dM=4
0.7"	140	137	200	180	169	187	193
1"	106	111	130	153	131	141	164
1.5"	81	95	93	121	134	121	210
2"	64	70	84	122	98	122	135
2.5"	66	70	61	84	79	104	119
3"	44	69	51	70	77	79	90
3.5"	47	53	75	80	93	101	76
4"	29	60	63	76	78	86	94

website and directly from USNO DVD
- Aladin Sky Atlas v8.0
- SIMBAD, VizieR
- 2MASS All Sky Catalog
- URAT1 Survey
- AstroPlanner v2.2
- MaxIm DL6 v6.08
- Astrometrica v4.10.1.432
- XLSTAT 19.02.42992

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Rule of Thumb Proposing the Size of Aperture Expected to be Sufficient to Resolve Double Stars ...

Appendix I

Table 4. Size of Central Disk and Encircled Energy Depending on Size of Central Obstruction (Mahajan – 2011)

Size of CO	0.00	0.10	0.20	0.30	0.40	0.50
Size of Central Disk	1.22	1.21	1.17	1.11	1.06	1
Encircled Energy	0.838	0.818	0.764	0.682	0.584	0.479

Table 5. Calculated Reduction Effect in mm on the Resolution for the Required Aperture for an Equal Bright Double with 1" Separation

Size of CO	0.00	0.10	0.20	0.30	0.40	0.50
Calculated effect in mm on required aperture	0.00	1.27	5.26	10.59	15.96	20.79

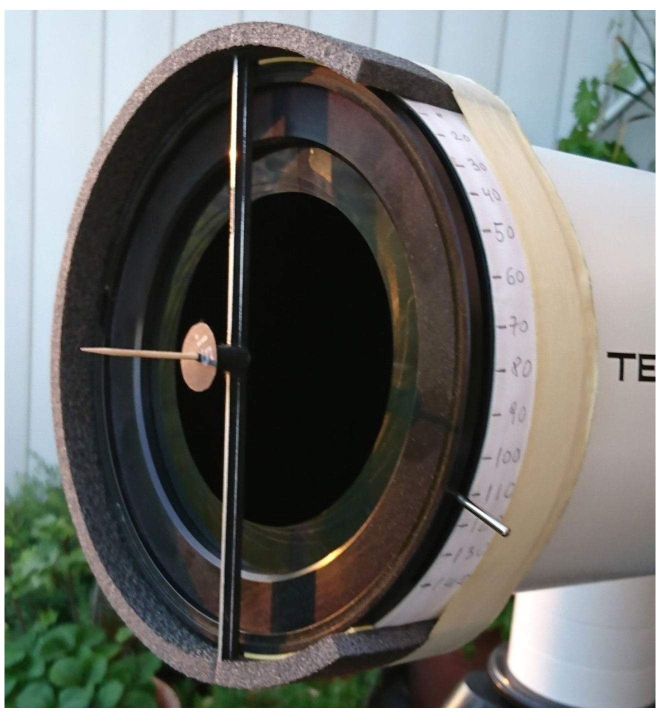

Figure 31. One of my telescopes with an iris diaphragm for changing aperture diameter plus a spider for applying a central obstruction of different size

Appendix II: Analysis of Outliers

To find an explanation for the objects with the largest differences between the result of the proposed RoT and the recorded limit aperture for resolution, I took a closer look at about 30 such objects. The hope to find a straight forward cause for these differences as, for example, color issues simply was not realized. The effort to take new images with V-and I-filter to be able to detect such color issues for at least some objects was in vain.

The most spectacular failure in this regard is ES 1906. I recorded for this object twice 140mm aperture (to be fair: not for a clear resolution but a rod) despite a magnitude of only1 13.58 for the secondary—more or less impossible to resolve at my location with 140mm aperture. I then suspected a color issue with a blue-white secondary, but images taken with I-filter suggested in comparison with V-filter images a rather reddish tint. So the ES 1906 outlier remains a puzzle and probably suggests rather two lucky hits. But also in most other cases no assumed color issues could be confirmed.

Some cases could be reduced to data errors leading to “wrong” RoT results, but this was certainly no general pattern as in most cases the WDS parameter used were confirmed by my own measurements or else the results of my measurements were used as input. In one case a simple typo was detected as cause for an outlier.

Yet one rather unexpectedly detected pattern is the fact that 2/3 of the counter-checked objects are listed with two or more observations and in most cases with some noticeable spread with usually one or more observation rather near the RoT result and at least one as outlier. So this suggests that different seeing conditions in different sessions are responsible for the recorded spread, but even this is not in all cases directly confirmed by my session logs with the caveat that the seeing was checked only at the beginning and might have changed during the session.

That different seeing conditions are with high probability the cause for large spreads and thus explain a good part of the outliers is in hindsight not really a surprise.